take place either to all members at fixed intervals of time, annually or otherwise, or only to each member at his death. The difference will not be great as regards any member. If, for instance, to each member a sum is annually credited proportional to the charge on premium paid by him in that year, but is suffered to accumulate in the Office till his death, at compound interest, it is much the same to him as if no sum were credited to him before his death, but that then a sum were given him (additional to the sum assured) proportional to the charges paid ever since he became a member, accumulated at compound interest: the difference of the two amounts would probably be very trifling. But as respects the security of the Office, the latter plan would be much more advantageous. In the former plan, the only security against unfavourable contingencies would be the charges on the premiums for a single year; in the latter, the security would be the whole accumulated amount of the charges of each present member, from the time of his entrance to the present time. And it will be generally the case, that the systems of division of profits in which the apportionment is made only at the deaths of the members afford greater security against casualties to the Office than those in which the division is periodically made, on whatever system it be made, and whether by way of bonus presently payable or of addition to the sum assured; or at least that a greater percentage ought in the latter case to be added to the net premium as a charge in order to give equal security.

On the Practical Application of the Doctrine of Chances, as it regards the Subdivision of Risks. By the late Dr. Thomas Young.

[Extracted from *The Quarterly Journal of Science.*]

It is well known to those who have studied the theory of chances, that where the magnitude of a risk may be divided into an indefinite number of parts, it is possible, by this multiplication of smaller parts, to confine the probability of the occurrence of any given excess or deficiency in the result, above or below its mean value, within any given limits; and it is of great practical importance to ascertain both how far the subdivision ought to be carried with regard to insurances, and what are the inconveniences to be apprehended from the admission of occasional deviations from the general rule.

We may take, for example, the case of an Office having under-

taken 1,000 risks of £5,000 each, at a fair premium of 1 per cent. per annum, so as to have an annual income of £50,000, to be expended in the payment of losses. It would be very inconvenient to such an Office to be liable to the frequent occurrence of an annual loss amounting to twice its income; and in fact the chance of such an occurrence is only about 1 in 333.

If the whole income of such a Society were derived from 500 risks of £10,000 each, the chance of a similar loss would be about 1 in 32, which is a degree of probability that ought not voluntarily to be incurred without some very powerful motives.

Still less would it be justifiable to engage the whole responsibility of such a Society in 100 risks only of £50,000 each; the chance of losing at least £100,000 in a year becoming, in this case, somewhat more than 1 in 4.

But it is equally demonstrable that a very exact adherence to the *precise amount* of the risk which is thought most eligible cannot be considered as essential to the reasonable security of the Society. Thus, admitting the propriety of confining the risks in general to 1,000 of £5,000 each; if a few risks (not more than ten, for instance) of double the amount were added to the number, the chance of losing an additional year's income, or £50,000, by the failure of five of them, would be no more than one in about five millions; if 20 double risks were accepted, the chance of losing five would still be only three in two millions; and if the same number of single risks were rendered more hazardous, so as to require a double percentage—that is, if 20 risks of £5,000 were accepted at 2 per cent.—the chance of losing a year's income by these would be next to nothing, and that of losing £30,000 would be only two in a million.

It may be observed, that the increase of the percentage on a risk which becomes more hazardous, on any given sum, brings with it an adequate remedy, in the increase of income, as well for the combinations of risks as for the single adventure, without the necessity of any diminution of the amount of the separate risks.

Taking, for example, the extreme case of a risk of cent. per cent. on ten adventures of £5,000 each; it is *certain* that the *whole* income will be lost, but impossible that the loss should at all *exceed* the income.

If the same income were derived from ten adventures of £50,000 at 10 per cent., there would be about one chance in three that nothing would be lost, and a little more than one in four that at least twice the income would be lost.

With 100 risks of £5,000 each, at 10 per cent., the chances of losing twice the income are only 1 in 505; the danger of exceeding the limit being less than one hundredth part of the danger incurred from the same number of risks of £50,000, at 1 per cent., and little more than half as great as the danger with 1,000 risks at 1 per cent.

It may also be inferred, from the examples here computed, that when an Office is well established and is prudently conducted there can be no practical necessity for having in readiness a deposit exceeding the amount of the annual income, which is supposed to be £50,000; it is also obvious that a similar sum would be amply sufficient for all the contingencies that are at all likely to occur in such an Office from the time of its foundation, during the gradual accumulation of its transactions, to the supposed extent.

The same mode of computation is equally applicable, whether the whole of the contingencies concerned are of the same or of different kinds, provided that the risks be fairly estimated; and the result would by no means be affected by their dissimilarity. There is a common prejudice, that it is disadvantageous for an Office to take a single risk of any particular description; and it is sometimes said, that if the adventure should happen to be unsuccessful, there would be no possible compensation from others of the *same* kind. There is, however, just the same chance that it would be compensated by others of a *different* kind; and if it were not, a fair price has been received for the responsibility, which it was worth while to incur from the probability of escaping, provided that the magnitude of the adventure was not too formidable; although the impression made *on the mind* by a singular event is sometimes stronger and more disagreeable than by a more ordinary one. No person has ever attempted to assign, by calculation, what the precise disadvantage is that attends on a singularity of risk; nor has it ever been defined what degree of singularity there must be in order to render a risk ineligible, except it involved a practical difficulty in appreciating it. It cannot be determined, for example, whether or no an insurance on the life of a negro, a mulatto, or an Indian, for an *adequate* premium, is a different *kind* of risk from an insurance on the life of a European; and if the affirmative were asserted, it would be difficult to show that a single insurance from a given town or county ought not to constitute an objectionable risk for a similar reason. And if it were possible to distinguish any imaginable diversity in the kinds of risks, so as to have, for example, 1,000

kinds; and if 1,000 equal risks of the separate kinds were at first undertaken by 1,000 different Offices—it is manifest that if each of these 1,000 Offices, instead of confining themselves to the same kinds of risks, exchanged, in the second place, one of its risks with each of the other 999, so as to have all heterogeneous instead of all homogeneous risks, the profits and losses of each Office being supposed to be fairly balanced on the original supposition, they would remain equally balanced under the new distribution, nor is there anything in the supposed change of combination that could affect the liability to greater or less deviations: at least, if there were any accident that could lead to such an inconvenience, it would probably be rather more likely to be diminished by the combination of heterogeneous elements than by the confining the separate Societies to their primitive homogeneous undertakings, which might possibly partake more of the nature of an undivided single risk in the liability to inconvenient fluctuations.

On a Method of Distributing the Surplus among the Assured in a Life Assurance Office. *By* T. B. Sprague, M.A., *Fellow of St. John's College, Cambridge.*

Suppose that the portion of surplus allotted to a particular policy at the division of profits, is applied so as to relieve the assured from the annual payments after arriving at an age to be determined by the amount of that portion or cash bonus, the other conditions of the policy remaining the same; or again, suppose that the cash bonus is so applied as to convert the assurance into an endowment assurance, payable at an age to be determined:—it is proposed to investigate formulæ to apply to these two cases.

In the first case, the cash bonus must be equal to the present value at the time of division of profits of a deferred annuity, of which the annual payments are equal to the premium for the assurance, the number of years for which the annuity is deferred being unknown; and the problem, in fact, is to find the number of years.

Let m be the age of the life assured at the time of the division of profits; P, the annual premium; C, the cash bonus; x, the number of years for which the annuity is deferred, which is at pre-